*All your algorithms must be written in pseudo code, and justified. A comparison will be considered as an elementary operation in O(1)*

Recall that a queue is any data structure supporting the following three operations:  
- **void** enqueue(**int**): add a new element to the structure  
- **bool** empty(**void**): tests whether the structure contains no element  
- **int** dequeue(**void**): removes and returns the eldest element added to the structure

1. Implement a queue by using two stacks. **/1**
2. Show that your implementation (or possibly, a slight modification of it) has amortized complexity O(1) for all its operations. – *Hint: consider the potential function P(n) = # elements in the first stack.* **/1**
3. Propose a similar implementation of a stack by using two queues. Is the amortized complexity of all its operations still a constant? Justify. **/1**

A *pancake sorting strategy* consists in sorting a vector by comparison, with the only allowed way to change the position of an element being to reverse all elements of a prefix subvector.   
Example: [1,4,3,2,5,6,0] 🡪 [6,5,2,3,4,1,0] 🡪 [0,1,4,3,2,5,6] 🡪 [4,1,0,3,2,5,6] 🡪 [2,3,0,1,4,5,6] 🡪 [3,2,0,1,4,5,6] 🡪 [1,0,2,3,4,5,6] 🡪 [0,1,2,3,4,5,6]

1. Propose an algorithm to reverse all elements of a prefix subvector in O(n) time. **/1**
2. Show that one can sort any vector with at most 2n reversals – *Hint: consider the largest element in the unsorted part of the vector*. **/1**
3. Give an example of a vector where Ω(n) reversals are needed. **/1**

Recall that the kth ordered statistic of a vector is its kth smallest element. You can use in what follows that such statistic can be computed in O(n) time.

1. The q-quantiles of a vector are its k\*n/q order statistics, for k = 1..q. Propose an algorithm in order to compute the q-quantiles in O(n\*log(q)) time. **/1**
2. Is this algorithm optimal? Justify. **/1**
3. Decide whether a vector encodes a binary heap in O(n) time. **/1**
4. Convert a min-heap into a max-heap in O(n) time. What implementation of a heap are you using for that? **/1**